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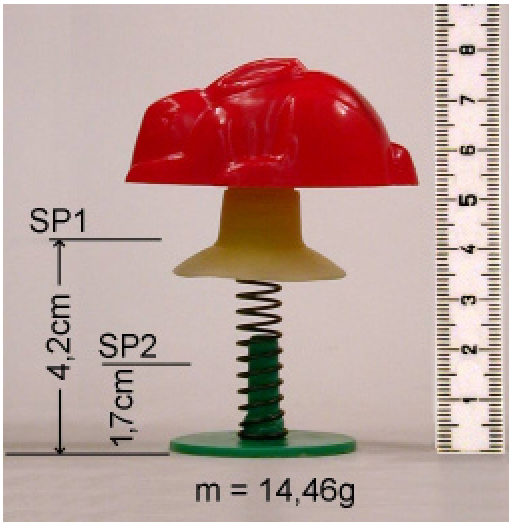
Jumping animals and toys

A flea jumps about half a meter high. At a distance of about d = 2mm it accelerates. This leads to an acceleration of $a = h \cdot g/d = 0{,}5m \cdot g/0.002m$ » $250g$ (g = $10ms^{-2}$; uniform acceleration assumed). Since the jumping height of a flea is strongly influenced by air resistance, it has, in reality, a greater acceleration. There are other animals with an even higher acceleration. A man can only achieve up to 3g with a standing high jump.



A small toy known as jumping animal or pop-up allows some investigations that can illuminate the physics of jumping. The toy itself consists of a base, a spring, a rubber cup and a head. You have to press the cup onto the base and thus load the spring. After some time the cup will loosen itself and the toy will jump up. The first experiment is, of course, to measure the jumping height. Sometimes you have to wait very long for the jump . With the same toy under the same conditions the height is about 1.20m and varies about 10%. The energy comes from the compressed spring, and you probably wonder how you can measure this. In a quick experiment you can press the toy on a simple balance. The result is about F ≈ 19N. The spring is compressed about d ≈ 3.5cm. Thus a spring constant of c » $19N/0.035m = 550Nm^{-1}$ *results.* A more accurate measurement is given with c = $500Nm^{-1}$ and d = 3.2cm. The energy stored in the spring *is* $E = 0.5 \cdot 500Nm^{-1} \cdot 0.032^2 m^2 = 0.26J$. The toy should reach a height of $h = E/mg = 0{,}26J/(10ms^{-2} 0.0145kg) = 1.79m$ (mass of the whole toy m = 0.0145kg). This is a great difference from reality. The explanation is seen if you remove the plastic head from the rubber cup. Several turns of the spring are pressed in the top of the rubber cup and can only move with great friction. Here mechanical energy is dissipated to heat.

Calculating the initial acceleration of the head of the toy is an interesting task: $a = F/m - g = 19N/0.01kg - g$ » $190g$. For the total mass only the sum of the mass of the head, the rubber cup and one third of the spring are used. This acceleration is quite good when compared with that of the flea.

If you assume a constant decrease in the acceleration from the beginning according to $a(y) = c(d-y)/m - g$, the final velocity of the head at the end of the acceleration phase is

$$v = \sqrt{2\int_0^d a\,dy} = \sqrt{\frac{cd^2}{m} - 2gd} \approx 7ms^{-1}$$

This can be verified only with some effort. With colleagues I made digital videos of the jump with 1000 and 2000 pictures per second [1]. The analysis of these pictures confirmed the calculated velocity.

Only from the videos another property of the toy could be derived. Immediately after the toy jumps up, the whole toy oscillates due to the spring. The frequency could be measured and was f ≈ 74Hz. The frequency can also be calculated

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$$f = \frac{1}{2p}\sqrt{\frac{c}{m}} = \frac{1}{2p}\sqrt{\frac{500Nm^{-1}}{0,0021kg}} = 78Hz \quad with\ m = \frac{m_1 \cdot m_2}{m_1 + m_2} \quad m_1 = 0,0089kg;\ m_2 = 0,00275kg$$

This small toy allows even more investigations if you vary the masses of the head or of the base. Children and physicists will also find out very quickly that they can shoot the base if they loosen the base from the spring. There is a certain danger involved.

[1] The video (AVI-File) can be downloaded under the URL:
http://www.e20.physik.tu-muenchen.de/~cucke/ftp/lectures/jump2.avi

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