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THE "META-PENDULUM'' OR : A SELF'-PROPELLED SWING

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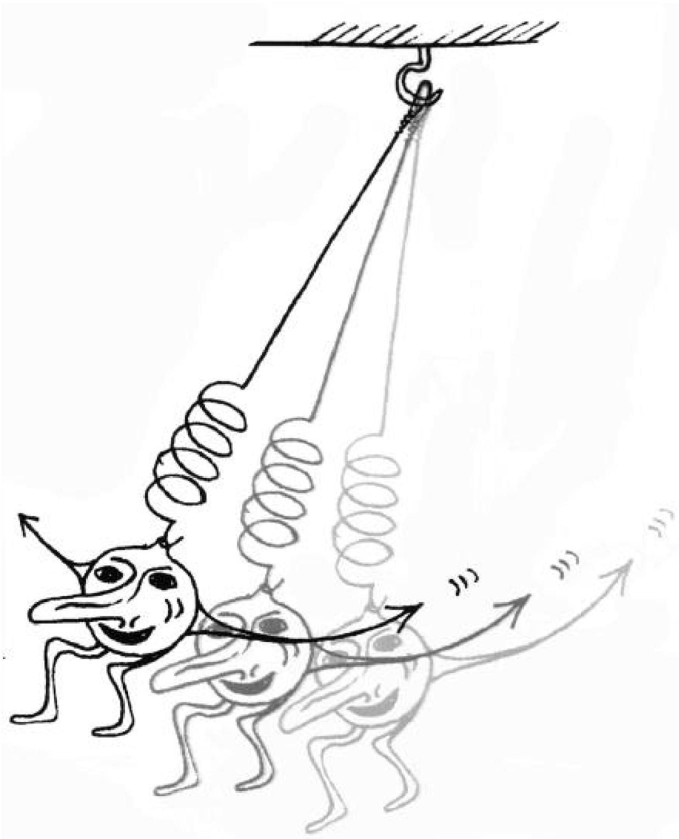
The world? An eternal swing!

Michel de Montaigne

The meta-pendulum does not only oscillate up and down and to and fro, but also, so to speak, in a meta-plane - to and fro between the two oscillation states.

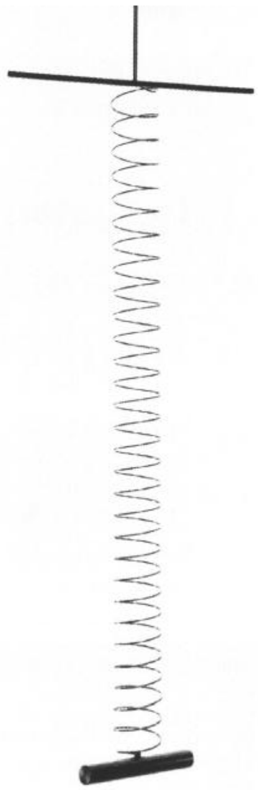


A steel ball is fastened at one end of a spiral spring which hangs by a thread from the ceiling in my room. There is scarcely a visitor who does not feel like pushing the ball downwards and setting it in vertical oscillations. Those who also dedicate a little attention to the result of this reflex action are soon greatly astounded. The pendulum behaves quite differently from what one naively expects: the up and down motion is strikingly strongly damped and at the same time switches to and fro motion which increases in intensity as much as the first one decreases. Finally nothing is left over of the original up and down motions the pendulum moves only to and fro. But before one becomes aware of it the vertical deflections of the spring start again and they finally as abruptly dominate as they disappeared. We can guess how it goes on. The switching back and forth between the vertical and horizontal oscillations continues as long as the energy of the oscillations gets dissipated and the pendulum stops moving. What fascinates here both those trained for physics and laymen is the precision with which the two kinds of oscillations switch between each other "at the expensed" of each other. However, the unavoidable energy losses due to friction cause the amplitude to get smaller and smaller until the pendulum finish itself at rest.

A person educated for physics will easily recognize in our meta-pendulum an example of parametric instability and non-linear resonance coupling as they are discussed nowadays in the realm of non-linear and plasma physics above all. Theoretically, we have to deal with a two-dimensional, mechanical problem which was solved in approximation by Minorsky [1] and presented in a simplified way by others [2, 3].

In the case under consideration, due to the oscillations of the spring, the parameter of the length of the pendulum varies periodically. However, such a process is unstable. Due to a non-linear coupling between both degrees of freedom of oscillations the ever present small disturbances are sufficient to deviate the pendulum horizontally and set it in swinging motion. Such a parametric excitation of the pendulum is similar to the propulsion of a children's swing. As the child in the swing rises and squats rhythmically and in phase with the swinging the centre of gravity of the system: child-swing moves periodically up and down, and the swing gets shorter and longer periodically. In this way the child sets the swing in motion and keeps it swinging. The difference between the meta-pendulum and the swing is that the child can use energy from the "inexhaustible" reservoir of its muscles and supplement the losses caused by friction. Thus the oscillations of the swing can be sustained as long as one likes.

The proceedings which lead to a horizontal pendulum are not reversible. In a general case the system remains in a state of to and fro oscillations chaotically modulatet up and down [4]. But, due to a suitable choice of the length of the pendulum, finally, a complete return transportation of the energy of oscillations follows. The pendulum must be heightened by a threat so that the horizontal oscillations of the pendulum get in resonance with the vertical oscillations of the spring. In this way the frequency with which the spring, due to the inertia of the oscillating ball, which, depending on the velocity is more or less deflected will be so tuned in to the proper eigenfrequency of the spring that the resonance condition is satisfied. A simple calculation shows [see Resonant Oscillations] that such a tuning in is reached if we choose the length of the pendulum numerically equal to the square of the period of oscillation (measured in meter per second) of the up and down oscillations. So, we only have to determine the period of oscillation of a spring pendulum which consists of a suitable pendulum body and a spiral spring. For this purpose we hang up this pendulum, set it in motion, measure the time of a number of oscillations and calculate the period of oscillations T_1. For example, if we get the value T_1= 1,12s the length of the pendulum must be $l_{ws} = T_l^2(m/s^2) = (1.12)^2 = 1.25m$. Now using a thread, we have to lengthen the system spring-mass, correspondingly - we measure the length to the centre of mass of the pendulum body. The meta-pendulum is ready.



According to our experience with springs commercially available the length l_{ws} of the pendulum turns out to be relatively uncritical, so the idealizations assumed above - the body on the pendulum as a mass-point , massless spring - are justified.

The attraction of our meta-pendulum, most of all consists in the fact that both kinds of oscillations switch from one to the other with the precision of a clock-work. When the initial conditions are changed and the displacement at the start consists of a "mixture" of vertical and horizontal displacements the behaviour of the pendulum is still regular, as the initial state repeats itself in a regular rhythm. But the clear to and fro and up and down states do not appear any more.

A similar shift between two different kinds of oscillations is obtainable with the so called Wilberforce pendulum [5, 6] (see corresponding figure). In this case the moment of inertia of the body of the pendulum is so adjusted that regular switch between vertical oscillations and rotational ones occurs. However the construction of such a rotational pendulum is a bit more complicated because here the pendulum body must be adjusted to the spring with respect to the form (moment of inertia) as well as to the mass. Form and mass of a solid body can hardly be varied independently of each other.

How would a pendulum, in which all the three kinds of oscillations were coupled, affect us?

Resonant oscillations:

Resonance between the vertical and horizontal oscillations occurs when the period of the up and down oscillations

(1) $$T_l = 2\pi\sqrt{m/k}$$

is exactly half of the period of the side oscillations

(2) $$T_\varphi = 2\pi\sqrt{l/g}$$

This gives

(3) $$T_\varphi = 2T_l$$

where m is the mass of the body of the pendulum, k the spring constant, l the length of loaded pendulum, g the acceleration of gravity.

Inserting (1) and (2) into (3) we get the length l_{ws} of the pendulum for alternating oscillations

(4) $$l_{ws} = 4gm/k$$

We get the spring constant from equation (1)

(5) $$k = 4\pi^2 m / T_e^2$$

Inserting (5) into (4) we get

$$l_{ws} = gT_l^2/\pi^2 = 0.994\,(m/s^2)T_l^2 \approx 1(m/s^2)T_l^2$$

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[6] http://valett.de/cv034.jpg