

Connecting past and future

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Abstract: *A huge number of excellent didactical approaches are known to exist in physics education; these treasures should not be forgotten but revived and combined with new developments and findings. Anamorphic pictures, known for more than 300 years, are fine examples involving some geometrical optics, some not too difficult mathematics and a lot of cultural background. These ideas can be explained in a completely traditional way, using conservative mathematical methods to calculate the anamorphic distortions, but today's powerful computer tools open additional new opportunities through modern raytracing methods from digitalized pictures. These enable the users to see almost at once their own pictures distorted and thus become motivated to vary parameters and to experiment for themselves. I will present this theme along with other examples and experiments and will discuss the physics and interdisciplinary background*

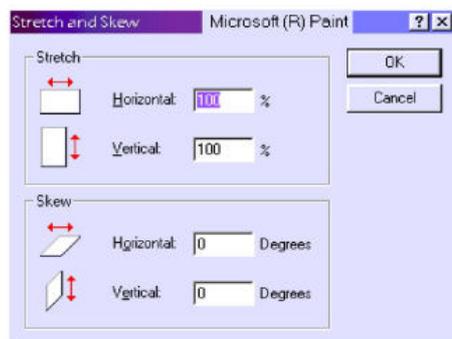
My goal is to present some ideas which will hopefully be fruitful for the users. Among my 'users' in Germany are physicists who would like to have some physics recreation in between normal physics work, teachers who can use some of these ideas for their lessons and students or pupils who can do experiments themselves and learn by doing. This group of users is a very broad spectrum. I will present some of my reflections and show you some experiments.

Let me start with the so-called anamorphic images or anamorphoses.

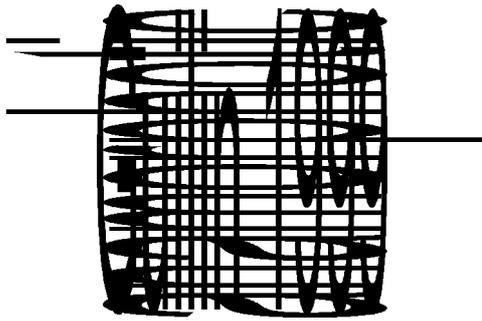
First Anamorphosis - Leonardo da Vinci,
Codex Atlanticus, 1483-1518, Milano



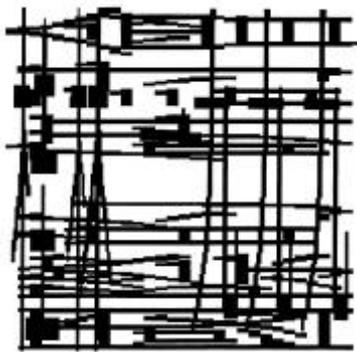
Anamorphic images are distorted images. You all know the fun children have in museums or at fairs when they see themselves distorted in big concave or convex mirrors. There are many types of anamorphic images. I will describe some of these here.



Probably the first known anamorphosis is from Leonardo da Vinci and shows the face of a child. If you view this image at a very shallow angle, you will recognize it. If you scan this image and have it in a digitalized form in your computer, you can 'work' with it. With many paint programs you can deskew and resize the image in all angles and directions and get a 'normal' image. When I look at this child's face, it could also be a Chinese child.



Here I have distorted some words in Latin letters. Try to recognize what is written here. It is possible but not clear at the first look. However, if you view this image at a very shallow angle, you will recognize it. First this view reveals 'Guilin 1999' at the bottom; then the view perpendicular to the first view reveals 'anamorphic'.



And here I have tried the same with Chinese characters. Please excuse my insufficient efforts with Chinese characters. But nevertheless it seems that this type of anamorphosis is also appropriate for Chinese letters. A Chinese colleague in my department said he was able to recognize at once what is written here. For non Chinese readers the characters mean 'Guilin 1999' and perpendicular 'International Conference'

This type of simple perspective anamorphic distortion can be find in many places: in paintings, wall paintings, drawings, underground stations or on the streets.

Hans Holbein: The Ambassadors (1533)
The national Gallery, London

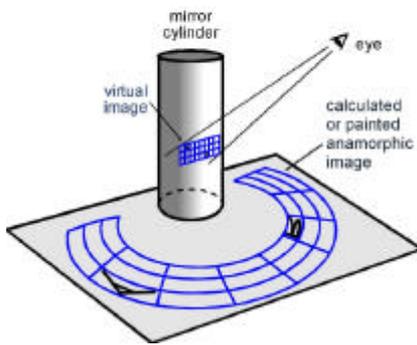


A very famous example is the painting 'The Ambassadors' by Holbein (1533), which shows two persons in a normal view. But there is a strange element. You can recognize what that is if you look at the painting in a very shallow angle. You can also deskew this part with some better painting programs like Photoshop or Paint Shop Pro. If you do that, you see at once that this strange part is a skull.

This type of anamorphosis was closely related with the development of perspective in the fifteenth and sixteenth centuries. And of course you can describe this in mathematical terms as a relatively simple form of linear transformations.



Cylindrical Anamorphosis



The so-called cylindrical anamorphoses were not developed until quite a time later. The pictures drawn appear correctly when observed with a cylindrical mirror (or conical or prismatic mirror).



Chinese Anamorphosis, 16th century

Mainly artists used this form of anamorphic pictures, not only in Europe but also in Chinese culture, and they used it without thinking in physics or mathematical terms. I will show here only one example of a Chinese anamorphosis from the sixteenth century. I have not found another one. The anamorphic picture is turned upside down. And here you see the solution: a pair of lovers. As far as I know, it is still not clear whether cylindrical anamorphoses were developed independently in China and Europe or whether one culture influenced the other. Since the Chinese culture has a special relationship to the topic mirror, it is at least probable that the Chinese developed anamorphic pictures independently. This is an interesting question.

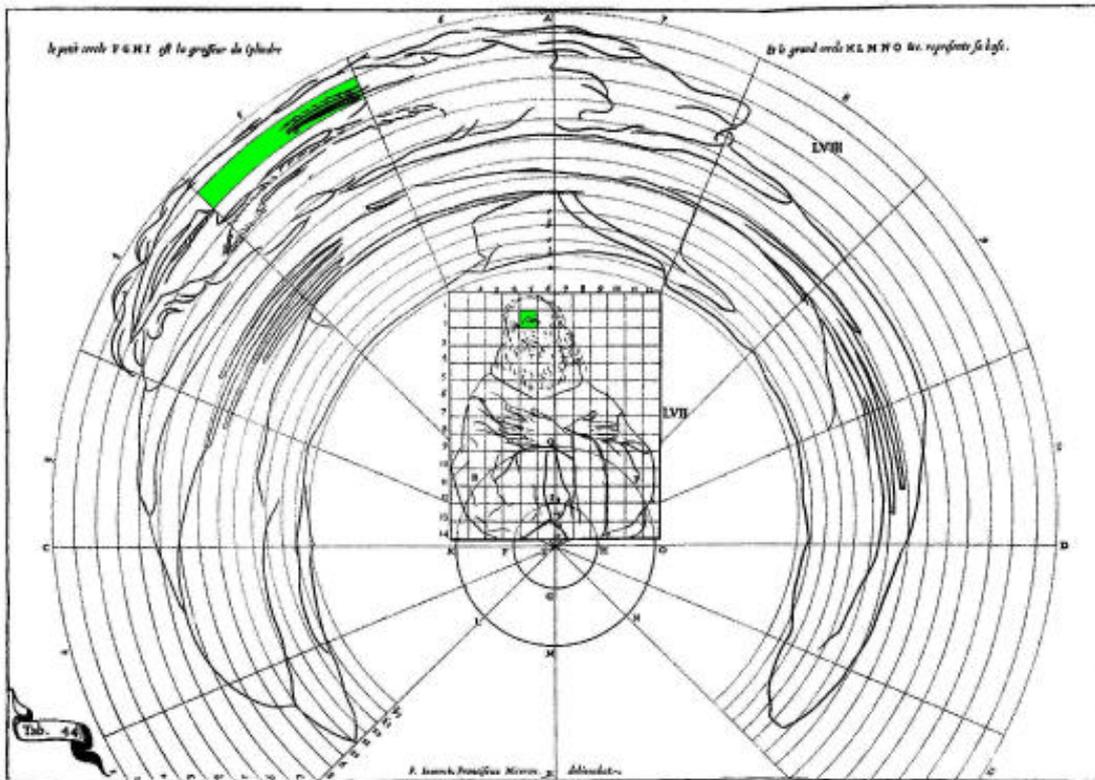


With anamorphic images you can hide the content. If you know the trick of how to resolve the image, you can see them normally. Therefore these images have a certain magic aspect. And this is fascinating, not only for children.



Look at this image from the Hungarian artist István Orosz. You see a landscape where some people seem to be looking at a shipwreck. You can guess that there must be something more because I am showing this in connection with anamorphic images. Put a cylindrical mirror on the moon (or sun?) and suddenly a face appears. The man is, by the way, Jules Verne.

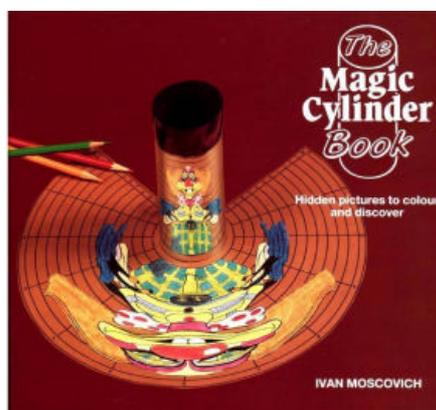
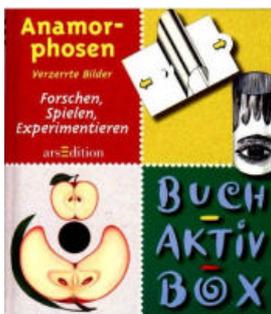
NICERON, Jean Francois: La Perspective Curieuse, Paris 1638



In Europe the French friar Niceron investigated the optics of anamorphic imaging systematically in the sixteenth century. This drawing about how to construct an anamorphic picture originates from him. It was a first attempt and is not quite correct with regard to the optics and mathematics. But that is not important, because the images which were constructed in this way are not very sensitive to small deviations.

Mathematically this is a transformation from a cartesian (rectangular) system to a polar coordinate system. Artists don't normally use this construction system. They paint free-hand.

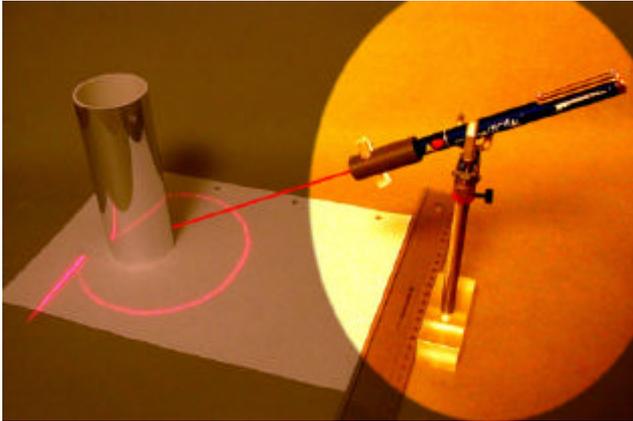
This structure from Niceron has been reproduced almost unmodified since then. You can find it in many books about anamorphic pictures. For instance in this small 'experimental



book' that is sold successfully in Germany and France. Or in other books (Magic Mirror) by the Hungarian inventor and author Moscovich. Moscovich also clearly explains the law of reflection for plane and curved mirrors.

In the internet there are several examples of how to use anamorphic art. They focus mainly on three subject areas: art, math and physics. In this way they show the connection between them.

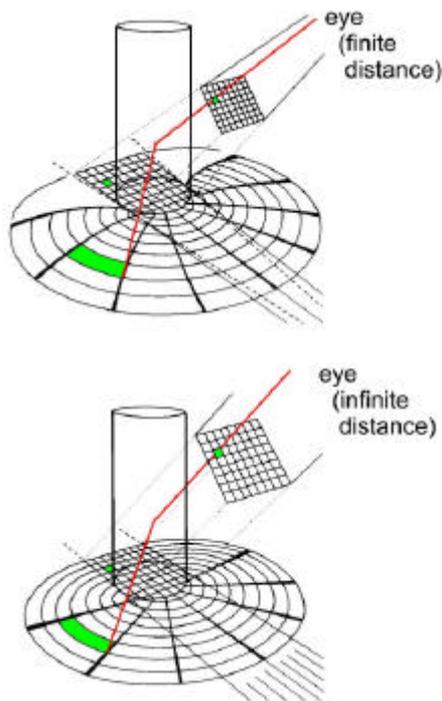
As I have already mentioned, the simple polar coordinate system is quite sufficient but not completely correct.



There is a simple experiment on how you can create a correct anamorphic grid. Take a laserpointer and make a construction where a cylindrical lens is fixed in front of the laserpointer (I have taken a small piece from a perspex rod from a curtain). The result is a straight laser line. The reflected laser line will hit the plane and thus directly mark one part of the anamorphic grid. Obtaining a complete grid in this way means quite some

work. This is, by the way, principally the raytracing method that I will explain later more in detail.

If you compare this grid with the polar coordinate system, you will see some specific differences especially in the corner regions. The correct grid does not consist of circles! And straight lines in the original object are not transformed into straight lines in the plane.

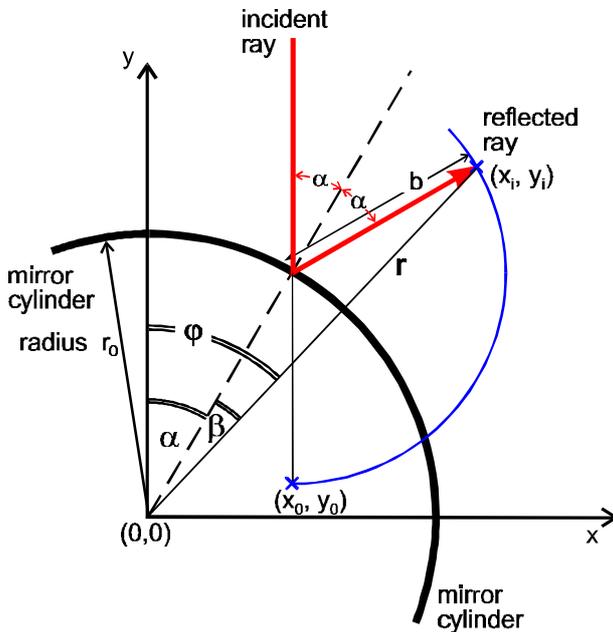


I will now try to go deeper into the mathematics and physics of the anamorphic transformation.

Here are two drawings about different possibilities of viewing or constructing anamorphic grids. The first is the normal one where the observer is at a finite distance from the cylindrical mirror. But this case is mathematically complicated. As you can see, even the vertical straight lines from the quadratic object grid are not transformed into straight lines in the anamorphic grid. One reflected ray is drawn in red.

In the other case the observer is at an infinite distance. In this case the vertical straight lines from the object grid are also transformed into straight lines in the anamorphic grid. This case is mathematically easier.

The difference between these two cases can be seen clearly. You can also see now why small deviations in the anamorphic grid will have an even smaller influence on the image. The whole anamorphic image is reduced into the image you see.

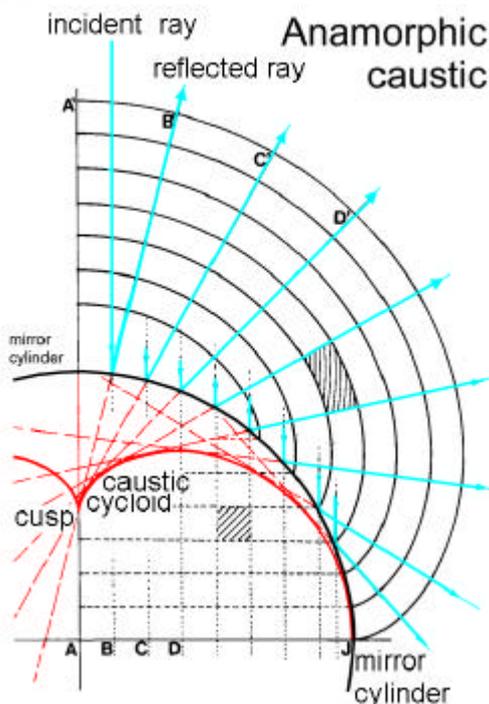


$$b = \sqrt{r_0^2 - x_0^2} - y_0$$

$$r = \sqrt{b^2 + r_0^2 + 2b\sqrt{r_0^2 - x_0^2}}$$

$$\varphi = \arcsin \frac{x_0}{r_0} + \arcsin \frac{b \cdot x_0}{r \cdot r_0}$$

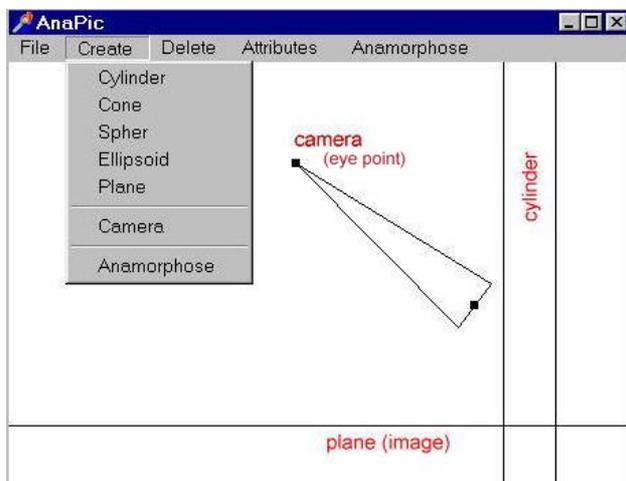
Now we are looking at the cylinder from above. Here are the incident and the reflected rays. The construction is simple with the law of reflection, which means that the angle between the incident ray and the perpendicular to the cylinder surface must be the same as the angle between the perpendicular and the reflected ray. Additionally, the distance between the reflection point on the cylinder surface and the point you want to calculate must be the same as the distance between the reflection point and the equivalent point in the object grid. With elementary mathematics (principally sine and cosine rule) you get the following equations for the polar angle and the radius. And this is not a linear and not even a quadratic transformation. One must be careful with the definition and handling of the polar angle and with the sign conventions for angles.



Some other interesting properties can also be seen. If you extend all the reflected rays backwards and then draw the tangential curve, you receive exactly the caustic that is known from the reflection in a convex mirror. This curve is a cycloid. A simple mechanical construction for the anamorphic grid results from this. Cut out this cycloid so that you get a cycloid with a thickness of perhaps 1cm, take a piece of string and attach one end onto the cusp. At the other end attach a pencil. Now you can draw the curved anamorphic grid lines directly by moving the pencil with the string tautly. Centuries ago machines were even developed to draw anamorphic pictures automatically.

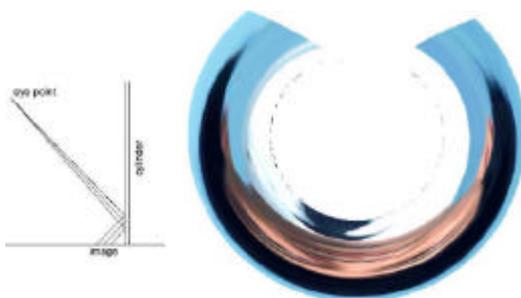
With the formulas it is clear that you can calculate any anamorphic picture pixel by pixel with a computer. I call this the classical way of calculation. Programs for that have been written. I did some myself about 15 years ago. Another program under DOS was written by the German student Juergen Bergauer in the Institute of Theoretical Mechanics at the University of Kaiserslautern in Germany, but the program is not easy to handle. I have looked for other easily available programs but I have not found anything in the internet or any other reference in literature. If any of you know a source, please let me know.

Another method of calculating anamorphic images is the so-called raytracing. In the case of raytracing anamorphic images, a virtual slide projector with the non-distorted, original image is assumed. From the eye a ray goes through the original image pixel by pixel and hits the mirror, where it is reflected and then hits the plane or another surface in a pixel. The color of the pixel is the same as in the image. The mathematics is principally similar to the classical method.



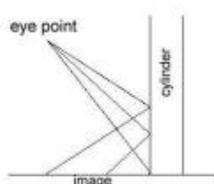
The German student Friedel Ulrich developed a program for anamorphic images with the raytracing method as a thesis during the college-level at his German Gymnasium in Pfaffenhofen in Bavaria. He even won a prize in the German competition 'Young Researchers'. The structure of his program can be seen from this transparency. A camera, a mirror object (cylinder, cone or sphere) and a plane can be selected and positioned. A digitalized image, e.g. from a digital camera, can be inserted. The program calculates the anamorphic image.

Both programs can be downloaded from the internet (URL: <http://www.e20.physik.tu-muenchen.de/~cucke/ftp/anamorph/>). There are short help-files in English. Both methods, classical calculation and modern raytracing, need a lot of computer power and therefore are not very quick, especially for high image resolution. With a Pentium 400MHz processor you need several minutes.



My last example - a cylindrical anamorphosis - was calculated with this raytracing anamorphic program. I have to remark that anamorphic images are normally turned upside down, which makes it even more difficult to recognize the content. The first image is very strongly distorted. I believe nobody can recognize which person is portrayed here. The second image is not di-

started as strongly. Now you may already guess who it is. Finally here is the solution: Prof. Luo XingKai. Please excuse me for taking your picture from the internet.



Prof. Luo XingKai



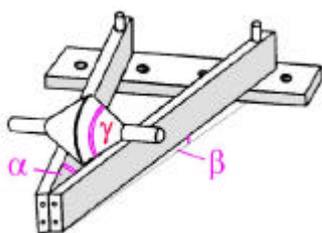
I want to cite one author here. In the publication 'Anamorphosis' (Mathematical Gazette 76 (1992), 208-221) Philip Hickin wrote about the use of anamorphosis in school classes: When I gave this activity (drawing cone anamorphosis) for homework I had the best response I have ever had from a class.

When you think about what I have told you so far, not much physics has been involved. This is an important point. I believe physics teachers must embed their lessons much more into other areas of more interest for the students, even if this means that 'real' physics takes only a minor part of the whole lecture. This is especially important for students who don't have physics as their principal interest.

Several difficulties will occur, strongly depending upon the level of teaching: A physics teacher is probably familiar with some non physics topics, but only up to a certain level. On a higher level the physics teacher cannot know these topics as well as an expert. So he should be more an organizer of the themes and the students. He must cooperate interdisciplinarily with colleagues.

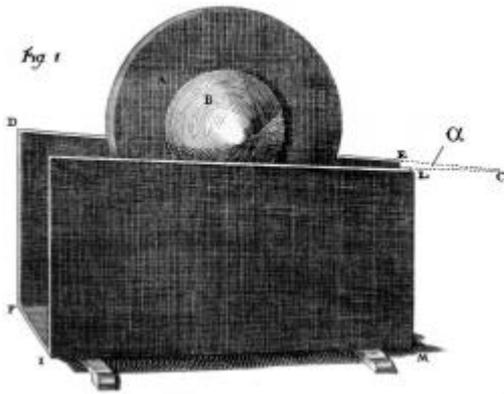
German physics teachers who I have encouraged to engage in the topic of anamorphic images have told me that they were successful with this approach.

The cone that rolls uphill An "antigravity" experiment



This is an old and well-known experiment. The principle is simple, but the performance is surprising. There is also a little magic for non-scientists.

A roller is made by uniting two congruent cones by their bases. It is placed on two inclined tracks meeting at an acute angle. The tracks have an inclination angle β to the horizontal. The doublecone will roll uphill under certain conditions. The center of gravity rolls downwards, of course



EXPERIMENTUM 10.
 Duo Plana IHL & FDE verticalia, ita disponuntur, ut angulum contineant; quare distantia EL minor est distantia DH; puncta autem D, H, magis elevata sunt quam E, L.
 Inter hæc Plana ponitur rota A, cujus axis B formatur ex duobus conis, quorum bases ipsi rotæ applicatæ sunt. Rota à lateribus DE, HL, Planorum sustinetur, & sponte DH versùs, ubi elevatio maxima est, tendit.
 Propter majorem inter Plana distantiam in DH, Rota A, cujus axis ab utràque parte est conus, magis descendit inter Plana, quando illam partem versùs movetur; idèò Gravitate suâ huc fertur, si modò descensus inter Plana superet ascensum, ex anguli HCD inclinatione ad horizontem oriundum.

This experiment is old, as you can see from this picture. For friends of Latin there is the original description. A friend of mine who is a physics teacher talked with his colleague for Latin, and then combined physics and Latin and had the pupils translate this small text. With some historical remarks you can make a small project out of this experiment.

PHYSICES
 ELEMENTA
 MATHEMATICA,
 EXPERIMENTIS CONFIRMATA;
Sive
 Introductio ad Philosophiam
 NEWTONIANAM.
Auctore
 GULIELMO JACOBO 'sGRAVESANDE.
 TOMUS PRIMUS.
Ediitæ Quarta, auctior & correctior.

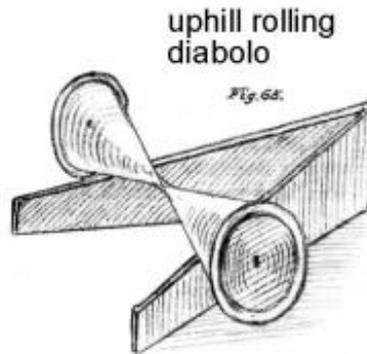
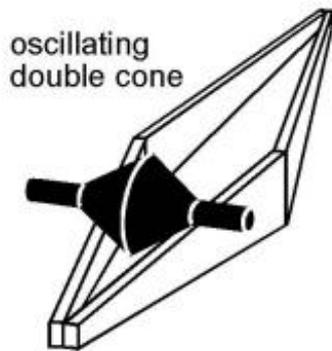
Here is the title page of the book by the Dutch author Gravesande, where this drawing comes from.

If α is the angle between the tracks and γ is the cone-angle, the condition for uprolling is

$$\tan\beta < \tan(\alpha/2) \cdot \tan(\gamma/2)$$

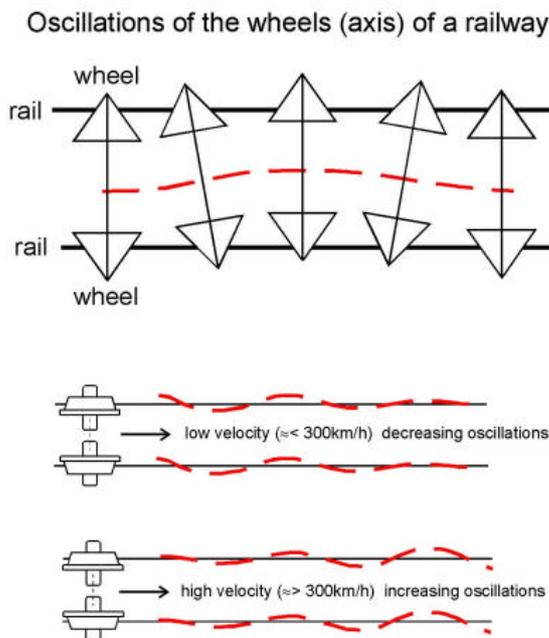
The proof of this condition is not very complicated but a little bit lengthy (see Ucke, C., Becker, J.: Roll Kegel roll!, Physik in unserer Zeit **28**, (1997), 161-163; in German). You have to look carefully at the relations between the different angles.





Here are two other possibilities to realize the experiment. With this special double track you get an oscillating double cone. And in this way you can realize the experiment with a diablo.

In this way the experiment is a nice hands-on experiment which can be used in classes. It shows the principle clearly, and it is easy to construct.



I want to mention an interesting property of the double cone. Railway engineers know it. If you set the cone not perpendicular to the center line of the tracks but a little obliquely and let it roll, the cone will wobble at first but then stabilize the line of the movement to the center line as you can see here. This also happens with parallel tracks. This behavior is very important. The wheels of railways also have a double cone profile but with a very small cone angle. The wheels will stabilize themselves. But this is dependent upon the velocity (and other parameters). It can cause instability at high velocities. It would be an interesting and probably complicated problem to treat this with the aid of theoretical mechanics.

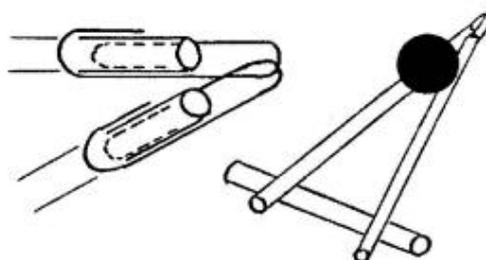
What about the quantitative acceleration and movement of the center of gravity of the double cone? This is also a somewhat complicated problem and appropriate for students of theoretical physics or engineering. This problem has been investigated in the Austrian university of Graz but not yet published.

The following game is fascinating, not only for children. And the background is the same as with the uphill-rolling double cone. My wife has suggested the name Sisyphus for this game because you always have to try the same frustrating procedure again. Sisyphus is an unfortunate man from Greek mythology who was punished by the old gods to keep on rolling the same stone up a steep hill.

Sisyphus



Construction with straws, paper-clips and marble
The Physics Teacher 16 (1978), page 16



This arrangement can be built out of two billiard queues and the billiard ball or even simply with two straws, paper clips and a marble.

Conclusion

These examples show clearly the actuality of long-known and proved physics experiments. They can be used on very different levels, from kindergarten to university, sometime amusing, sometime serious. There are even some problems which have not been solved or published. By using the power of modern technical aids (e.g. computers, video camera) you can find new approaches. There are strong connections with other disciplines, which is probably a very important point nowadays. Teachers are encouraged to cooperate with colleagues in other disciplines. Intercultural questions are also involved. I know other examples where similar approaches can be used and that it is therefore fruitful to look for them. This is an everlasting challenge.