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Paradoxical Shadows

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Abstract
A bright source of light (sun, flashlight with a bright parallel beam) can, under certain circumstances, simultaneously create several shadows beneath a ball floating in water. Although this so-called shadow-sausage effect was published in 1967, it is, nevertheless, not well-known, especially not in connection with a spherical object.

If we were on a planet circling around a system with two suns, we wouldn't be surprised to see two shadows behind one object. Such actually inhospitable planets (Kepler-16b and Kepler-34b, for example) do, in fact, exist. In the science-fiction saga "Star Wars" with Luke Skywalker, this kind of planet was portrayed, albeit as one supportive to life, and given the name of Tatooine.

On our planet Earth, however, we become confused and curious when we see as many as three shadows under a ball floating in a shallow wading pool for children which is shone on at a sharp angle by one sun (figure 1).

fig.1: A ball shone on by the sun with a three-fold shadow.

In our experiment, a sphere-shaped plastic ball (Ø = 5.5cm) is floating about half immerged in 9cm-deep water. The light of the sun hitting it at a sharp angle creates the shadows reproduced in figure 1. At the edge of the sphere, a concave meniscus of the water surface is formed, as can be recognized by the distortion of the shadow line. From the figure, the width of the meniscus is estimated at about 0.4cm. The exact profile of the meniscus depends on the surface tension and the wetting properties of the sphere.

In figure 2, two cross-sections of the situation are represented, one from the side (on the left) and one perpendicular to it (on the right). A ray of light goes past on the left side at the edge of the sphere and strikes the meniscus. It is refracted more strongly towards the perpendicular (solid line) than a ray of light falling on the flat water surface (dotted line). A ray of

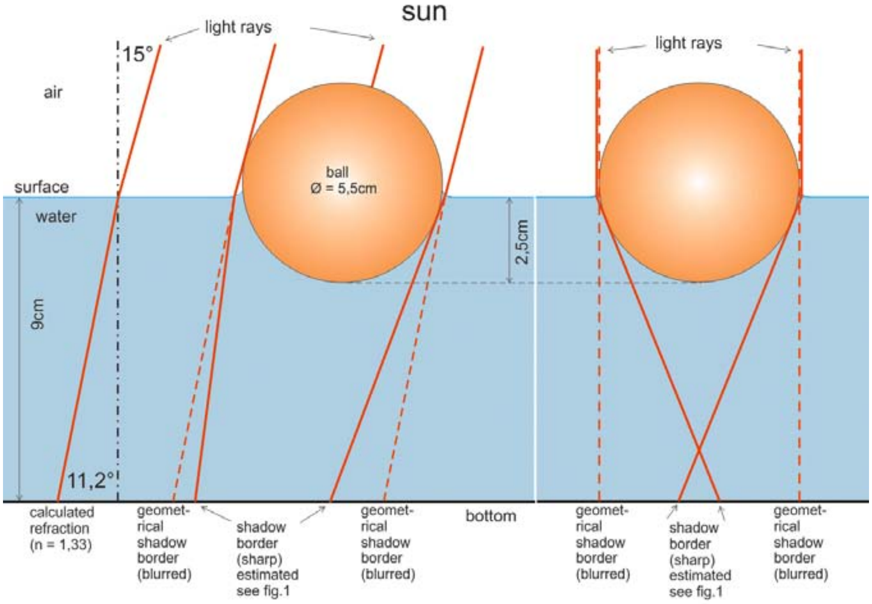

fig.2: Two cross-sections perpendicular to one another showing the situation represented in fig. 1. On the left, the rays of light disperse in the drawing layer; on the right, they are inclined 15° against the drawing layer. Dimensions are true to scale. The size and form of the meniscus and the refraction on it are estimated.

light on the right side of the sphere which directly hits the surface of the water at the edge of the sphere on the upper edge of the meniscus is refracted towards the sphere and absorbed on the surface of the sphere. Only a ray of light which hits the meniscus at a certain distance from the sphere is refracted towards the sphere and passes by beneath the surface of the water at the edge of the sphere (solid line). A ray of light falling on the flat surface of the water (dotted line) is refracted less strongly.

If we look at a cross-section rotated 90° towards us (figure 2 on the right), the situation is somewhat different. Now, a ray of light passing by the left or right side of the sphere hits the meniscus surface closer to the sphere. Then, due to the larger angle of incidence of the meniscus surface of the water, the deflection takes place strongly towards the middle. This stronger deflection causes the constriction in the middle of the relatively sharp-edged double shadow. The thickness of this constriction is extremely dependent on the depth of the water, as can be deduced immediately from the figure. Brighter parts may also be visible in this constriction. This is caused by caustic-like light apparitions resulting from superimposition of the refracted rays at differently inclined areas of the meniscus.

Fragmentation of the rays by the meniscus creates brightening of the shadow and fuzziness of its edge, which would result from a purely geometric observation without consideration of the fragmentation at the meniscus. That is the big, blurry shadow in figure 1 whose diameter corresponds approximately to the diameter of the ball.

The forms and sizes of the shadows also change greatly depending upon the immersion depth of the sphere in the water. If, for example, the sphere is only immersed very little, the rays pass by the edge of the sphere, but they do not reach the meniscus. For very deeply immerged spheres, the rays are broken at the meniscus, but then hit the sphere and are absorbed in it. The inclination of the rays against the perpendicular also plays a role.

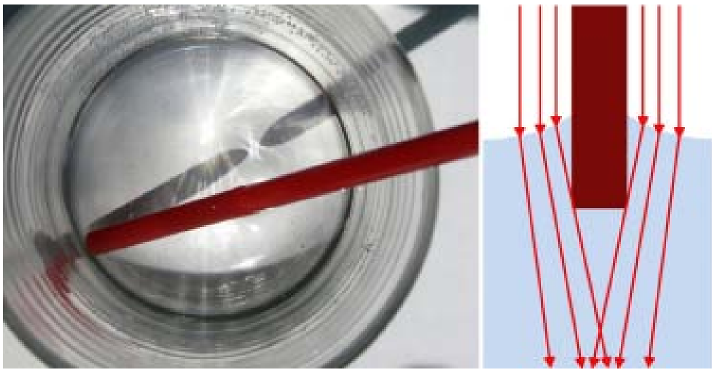

fig. 3: shadow-sausage effect with a round pencil.

The basis for this apparition is the so-called “shadow-sausage effect”, which was described in 1967 [1]. The author of the original publication on this effect placed a round pencil in a slanted position into a container full of water under strong light. The shadow on the bottom of the container showed a constriction which looked like sausages linked to one another (figure 3). The meniscus on the edge of the immerged pencil is responsible for this effect. Meanwhile, more authors have dealt with this in mathematically challenging publications [2, 3]. Their focus, however, is more on the caustic-like apparitions.

The experiment can easily be put into practice by floating a ball in water. Wooden or plastic balls with a diameter of several centimeters which are about halfway immersed in the water work well. A concave meniscus must be formed when the ball is made wet. A tennis ball can also be made sufficiently wet, despite its felt surface. In dim enough surroundings, a bright flashlight is also sufficient for generation of a three-fold shadow on the bottom of a container full of water. The depth of the water may have to be adjusted. The experiment with the pencil can be performed even more quickly.

keywords
shadow, shadow-sausage effect, light refraction, caustic, surface tension, water meniscus

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