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The Magnetic Cannon

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If you place a row of several steel balls on a track and let another ball strike it at one end of the row, then the last ball at the other end will fly off, corresponding to the incoming impulse. All the other balls will remain at rest. This experiment is well known from the Newton's Cradle toy. However you would be very surprised if the last ball in the row shot away like a bullet, even though the first ball only seemed to have a very moderate velocity. This would not seem consistent with the law of the conservation of energy.

The effect is so surprising that one guesses it might be a trick, and you would be right: one or more of the similar looking balls are in fact magnetic (fig. 1).

The first strike, approaching from the left hand side, is from a non-magnetic steel ball which will be strongly accelerated through the magnetic ball and thrusts with a relatively high velocity on the row of resting balls. This strong acceleration and high velocity only occurs in the last few milliseconds and is almost non- observable. The steel balls on the right hand side of the row are of course also attracted to the magnetic ball but the attraction of the last ball is not as strong. Therefore the impulse of the impacting ball is sufficient to liberate the last ball on the right hand side from the magnetic field emanating from the magnetic ball. This last ball will shoot away very fast, according to velocity of the incoming ball [1, 2, 3]. An even higher velocity can be achieved if the approaching ball from the left hand side is also magnetic – the magnetic attraction of two magnetic balls is almost double that of one steel and one magnetic ball.

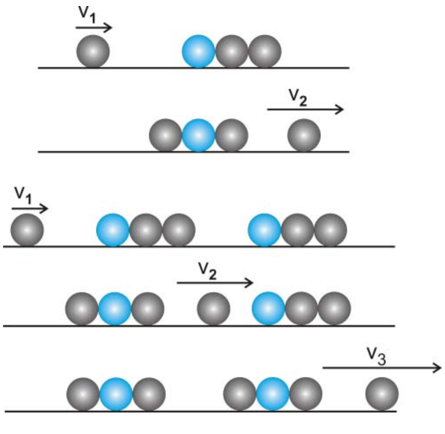

Fig. 1: In the top two rows a steel ball (grey) strikes a magnetic ball (blue). In the bottom three rows a steel ball causes a cascade of collisions.

Fig.2: A magnetic cannon sold as a toy

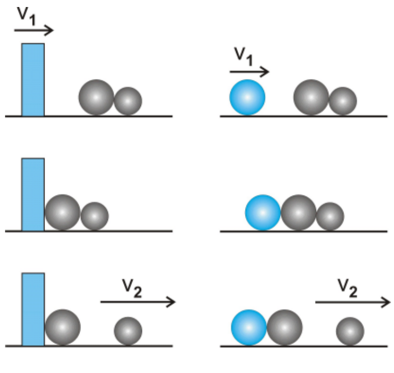

Fig.3: Variations of the magnetic cannon

A further increase in velocity occurs if you arrange several groups of balls, and in this way create a cascade of collisions (fig. 1, bottom part). The fast ball from the first strike will be attracted and thus accelerated a second time. A continuation of this arrangement is conceivable and reminds one of large particle accelerators. Which maximal velocity can be achieved thus is not yet tested, according to our knowledge.

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A toy with the name Gaussian gun or Gaussian cannon is already available, as shown in figure 2 [4]. In this case all the balls are non magnetic but one or two cylindrical magnets are fixed to the track. The steel ball approaching from the left is accelerated towards these magnets and carries forward this impulse to the steel balls on the right hand side. The steel ball on the right hand side shoots away at a high velocity.

Another publication describes the situation of moving a magnet towards two steel balls of different sizes, which remain in contact within the magnetic field of the magnet (fig. 3). The larger impulse of the bigger steel ball will be directed towards the smaller ball (elastic collision) and thus cause it to shoot away. A similar effect is achieved with one magnetic ball and two steel balls of different sizes. Further variations with balls of other materials (e.g. glass) are possible.

If a magnetic ball rolls, its magnetic field moves with the ball. One can imagine the magnetic field of a magnetic ball being similar to the magnetic field of the earth. At a certain distance away from a steel ball, one of the two poles of the magnetic ball will orientate itself to the steel ball, at which point the magnetic ball will no longer roll, but slide.

Force laws of the magnetic cannon

If one wants to calculate the velocities and accelerations of the balls, one needs the force of a magnetic ball on another ball as a function of the distance. This function of course depends on whether one magnetic ball acts on another magnetic ball or on a steel ball. In some cases one can describe this dependency with a power law.

Fig. 4: Measurement of the holding force.

The measurements can be made with very simple resources. Figure 4 shows a magnetic ball on a nonmagnetic material (such as brass or aluminium) of a defined thickness. Underneath this is placed another ball (steel or magnetic) which is attracted to the upper ball. A small holder made from a nonmagnetic material is provided with a drilled hole, so that a sufficient part of the bottom of the ball fits through it, and maintains contact with the material of defined thickness from the bottom. Onto this holder, weights of increasing size can be attached, until the ball with the holder falls down. Thus the holding force can be measured.

Figure 5 shows the results of the force as a function of the distance from a magnetic ball to a magnetic ball (black line), from a magnetic ball to a steel ball (red line) and from a magnetic ball to a steel ball behind another steel ball (blue line). Since the measurement range covers almost three decades, a double logarithmic scale is recommended.

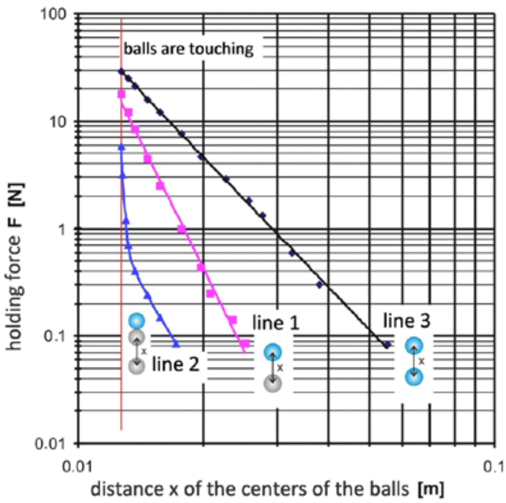

Fig. 5: Holding force F between balls as a function of their distance x.

The force F of a magnetic ball on a steel ball as a function of distance x (line 1) can be well described with the power law $F(x) = 1.934 \cdot 10^{-14} \cdot x^{-7.852}$ (F in N, x in m). A power of eight is rarely found among the laws of physics, and probably is the result of the dipole of the magnetic ball inducing a quadrupole in the steel ball. From the mass of the steel ball (m_s = 8.4g), the velocity and acceleration of the steel ball can be calculated.

The force F of a magnetic ball on a magnetic ball as a function of distance x (line 3) can be described with the power law $F(x) = 6.41 \cdot 10^{-14} \cdot x^{-4.0365}$ (F in N, x in m). A power of four results from the dipole of the magnetic ball which is situated in the magnetic field of another dipole. The magnetic ball forms a magnetic dipole with its magnetic field along the axis of the dipole. The relationship between the strength and the distance is a cubic function.

With these force laws we can try to determine the velocity of the balls. For that purpose we have slightly changed the arrangement of the upper part of figure 2. If all the balls are loose, a ball approaching from the left side will attract the whole row with the consequence that both elements will move together. Furthermore there is a kickback to the row of three balls when the ball on the right hand side gets the impulse from the incoming ball and shoots away. This can be seen very well in a simulation [7]. Due to the fact that with this arrangement there are additional frictional losses, which cannot be easily calculated, we have fixed the magnetic ball in the row of three balls with a nonmagnetic clamp (fig. 6). All the balls run in a plastic halfpipe with an inner diameter that is slightly larger than the diameter of the balls. Such halfpipes can be easily made from insulating tubes such as those used in electrotechnics.

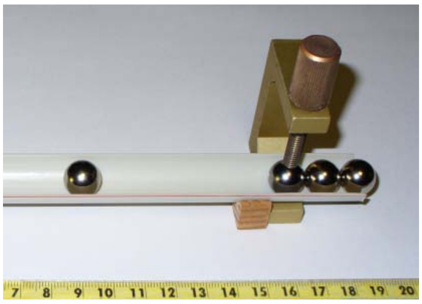

Fig. 6: Magnetic and steel balls in a plastic track. The magnetic ball in the row is fixed with a nonmagnetic clamp

In figure 7, the measurements of the force F as a function of the distance x of a magnetic ball on a steel ball, and a magnetic ball on a steel ball behind another steel ball (fig. 5) are shown in a linear diagram. The area between line 1 and the abscissa is the energy (28.7 mJ), which the ball approaching from the left side gains from the magnetic field of the magnetic ball. The area between line 2 and the abscissa is the energy (2.1 mJ), which the ball that is shot away from the right side needs to liberate it from the magnetic field of the row of balls. This energy is much lower because the steel ball on the right side is attracted with a relatively small force to the steel ball next to the magnetic ball. The difference of both energies is the energy which the ball that is shot away obtains under ideal conditions. You can then calculate the velocity:

$$v_e = \sqrt{2E/m} = \sqrt{2 \cdot 0.0266J / 0.0084kg} = 2.5m/s$$

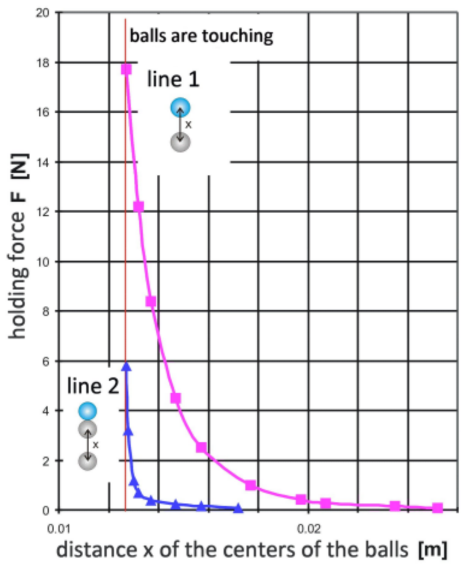

Fig. 7: Holding force F between balls as a function of their distance x.

Losses through friction and rolling, through non ideal collisions and perhaps also through eddy currents, are neglected here.

The velocity of the ball that shoots away can be determined according to a proposal from Kagan [3; fig. 8] by measuring the width of flying, *w*, of the ball, if height *h* of the falling ball is known

$$v = \sqrt{\frac{g \cdot w^2}{2h}}$$

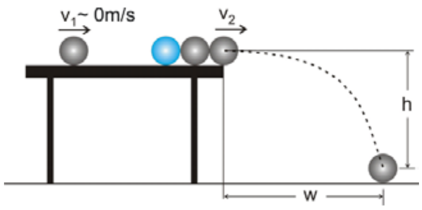

Fig. 8: Experimental setup for the determination of the velocity of the ball shot away.

We thus determine $v \approx 2.2$m/s. This is less than calculated with the force laws, but satisfying with regards to the unconsidered losses.

With the force law, one can also calculate the velocity of the ball approaching from the left side. In figure 9, the result is shown for a steel ball, which is approaching a magnetic ball without friction losses. The steel ball is positioned at a distance of about 3cm from the magnetic ball and then released. The ball is attracted and starts to move very slowly. Finally, in the last hundredth seconds, the velocity increases greatly. This can almost not be seen by the naked eye. This is the main reason for the surprising effect of the whole arrangement. The steel ball hits the magnetic ball with a velocity of about 2.5m/s, which is comparable to the previous results.

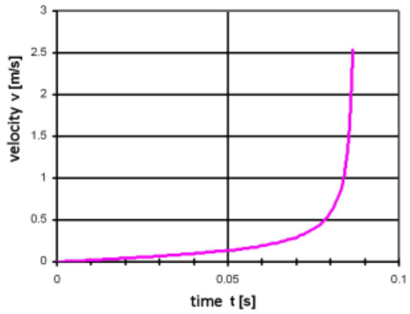

Fig. 9: Calculated velocity v of the approaching ball as a function of time t.

For the experiments we used steel and magnetic balls with a diameter of 12.7mm from the Swiss company supermagnete (www.supermagnete.com; also in English and other languages). These steel balls did not all have the same elastic properties. so we selected those with the highest elastic properties.

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Differential equation

A steel ball is attracted to a magnetic ball, which is fixed with a clamp. The attraction force F between these two balls as function of their distance can be described according to experiments with the power law

$$F(x) = c' \cdot x^n$$

F = force (in N); x = distance of the centres of the balls (in m);
c’ = constant = $1.934 \cdot 10^{-14}$ N;. n = -7.852.

What is the velocity of the approaching ball as a function of time?
The ball will accelerate according to Newton’s law $F(x) = m \cdot \ddot{x}$. The velocity will increase it comes in contact with the balls. We assume that the ball slides without friction. In reality the ball will roll at the beginning and then slide when close enough to the magnetic ball.
The ball starts at a defined distance of x(0) = 0.03m with no starting velocity $\dot{x}(0) = 0$.
The mass of the ball is m = 0.0084kg.
One can then write

$$F(x(t)) = -m \cdot \ddot{x}(t) = c' \cdot x(t)^n \qquad with\ x(0) = 0.03m\ and\ \dot{x}(0) = 0\,;$$

or $$\ddot{x}(t) = -c'/m \cdot x(t)^n \ = -c \cdot x(t)^n$$

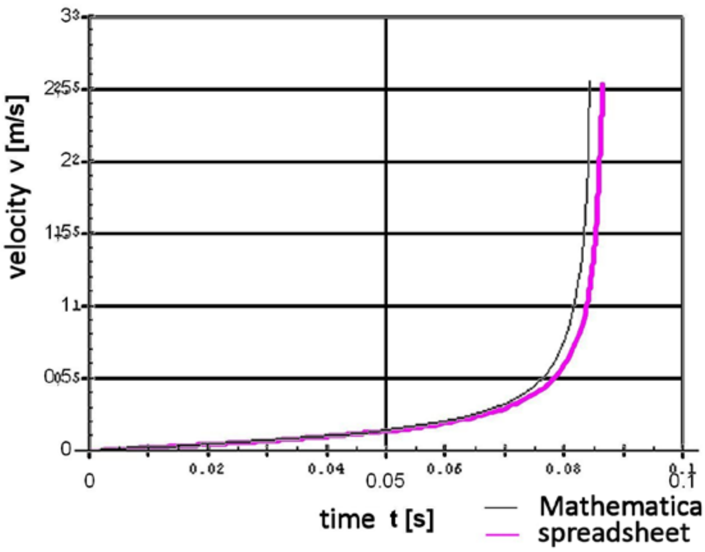
with c = $1.934 \cdot 10^{-14}$ N/0.0084kg = $2.3024 \cdot 10^{-12}$m/s²

This differential equation has, according to Mathematica (and an expert, whom I asked), no analytical solution.

A numerical solution is possible and is drawn in the figure below (black line).
After t = 0.0844s the ball has a final velocity of v = 2.567m/s.
The final acceleration is a = 1782m/s².

It is also possible to calculate the position and the velocity as a function of the time with a spreadsheet (e.g. Excel; red line), if a program such as Mathematica is not available.
The ball is in a defined starting position (e.g. x(0) = 0.03m). According to *F = m·a* the ball will be accelerated. The velocity increases by *a*Δt*. With that you can calculate the new position *v*Δt*. This leads to a new acceleration because the ball is closer to the magnet.

The results are similar to the Mathematica solution.



After t_e = 0.08643s, a final velocity of 2.57m/s and final acceleration of 1780m/s² is achieved.

The difference between both calculations is due to the finite number of incremental steps in spreadsheets.