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Back somersaults with jumping toy animals

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So-called somersaulting toys are small articles sold in toy stores that can do one or more back somersaults from a standing-position. There are two different versions of them that I am familiar with. The Somersaulting Frog (figure 1) has inside a spring that is pressed down and fixed to the ground by a suction cup. After a short while, the suction cup comes loose, and the energy stored in the spring is sufficient for a complete backflip. The second version (a kangaroo, gorilla, or mouse from the Japanese company TOMY) has a spring-winding tension that, in turn, tensions a second spring that is then suddenly slackened (figure 2). This toy can make up to seven backflips, one after the other. I will analyze the second kind of toy using the kangaroo as my example and then comparing it with a back somersault performed by a human. All my observations are primarily estimates and approximate calculations since more exact analysis would demand much more time and effort and not bring much additional knowledge or insight.

Fig. 1: Somersaulting frog.

Fig. 2: Somersaulting animal toys with a wind-up spring.

First of all, I made a film of some of the kangaroo's flips using a video camera with a high-speed shutter of 1/2000s. Figure 3 shows the series of nine pictures scanned into the computer.

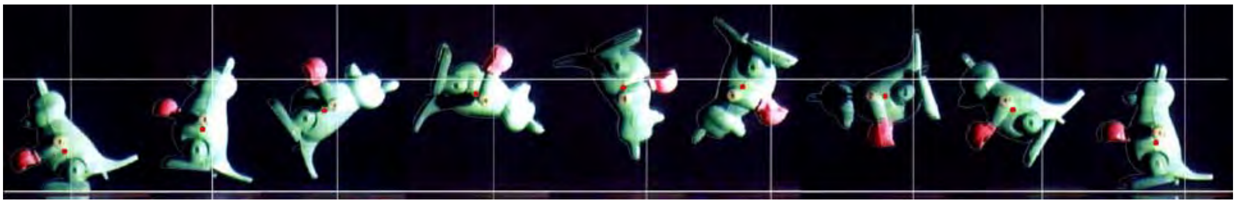

Fig. 3: Series of video images of a back somersault (kangaroo). The images have a time lapse of 1/50s (video half-frame lapse). The horizontal lines correspond to a distance of 5cm; the red dot on the green figure marks the approximate center of gravity.

By adjustment, the center of gravity was roughly localized and marked with felt-tip pen on the surface of the figure. This cannot be done very precisely since the kangaroo has asymmetric mass distribution in relation to the symmetrical axis of its body, i.e. the axes of its inertia ellipsoid are lopsided to the axes of its body. This makes the kangaroo teeter when it rotates. That is why a point marked on the surface of its body does not give an exact representation of the – parabolic – flight pattern of the center of gravity. The kangaroo is located behind a transparent foil with a graduation of 0.5cm. This makes it possible to estimate that the kangaroo's center of gravity rises to a height of *h = 3cm* above the ground. With a kangaroo mass of *m = 12g*, the resulting *potential energy* is:

$$E_{pot} = m \cdot g \cdot h = 0.012kg \cdot 10ms^2 \cdot 0.03m = 3.6 \cdot 10^{-3}J$$

In order to calculate the rotation energy, we need the inertia moment *I* and the angular velocity ω. Since the time lapse between the images is 1/50s (half-frame in the video camera), a reasonable angular velocity ω can be extracted. A complete back somersault, i.e. a rotation of 2π, extends through 8 images, resulting in:

$$\omega = \frac{2\pi}{\frac{8}{50}} s^{-1} = 39s^{-1}$$

For experimental determination of the moment of inertia, a small turntable was constructed using a spring that had been removed from a different somersaulting toy's defective wind-up spring. The result from measurement of the periods of oscillation with familiar and calculable objects (cylinders) and the kangaroo itself is:

$$I_{Kexp} = 2.6 \cdot 10^{-6}kgm^2$$

This can be estimated very roughly by regarding the kangaroo as a rod with a length of *l = 5cm*. The result calculated is then:

$$I_{Kexp} = \frac{m \cdot l^2}{12} = 2.5 \cdot 10^{-6}kgm^2$$

An estimation like this is, however, very sensitive, and it is probably more or less coincidence that the result calculated corresponds so well with the value measured. The kangaroo's moment of inertia remains constant during the rotation, in contrast to the human's moment of inertia. The values determined result in rotation energy at:

$$E_{rot} = \frac{I \cdot \omega^2}{2} = 2.5 \cdot 10^{-6} \cdot \frac{39^2}{2}J = 1.9 \cdot 10^{-3}J$$

This means that there is only about half as much energy in the rotation as in the potential energy. The total amount of energy is:

$$E = E_{pot} + E_{rot} = 5.5 \cdot 10^{-3} J$$

If the kangaroo is taken apart, measurements can be performed on the actual coil spring in order to infer the energy stored in the spring for a jump from the deflection and determination of the spring constants. The measurements contain large inaccuracies, however, since the length variation of the spring is only about 4mm. The amount of energy stored in the coil spring is about 0.01J. This is almost twice as much as the total amount of energy just calculated from position and rotation. There is clearly room for a discussion of loss of friction.

I do not recommend taking the somersaulting toy apart. On the one hand, this can easily damage it. On the other hand, the toy usually doesn't function correctly after being put back together.

Additional measurements can, however, be made without taking the animal apart. If the kangaroo is pressed carefully onto a scale that is sensitive enough to measure the jumping power of its legs, the resulting value is about *F = 0.9N*, almost independently of the angle of its legs in relation to its body. The acceleration distance for the jump resulting from this force can be calculated to be about *s = 0.8cm*.

If we assume that the acceleration vertically upwards affects precisely the center of gravity, the kangaroo's acceleration *a* is calculated to be:

$$a = \frac{F}{m} - g = \frac{0.9N}{0.012kg} - 10ms^{-2} = 65ms^{-2}$$

With the acceleration distance, the time *t* during which the kangaroo is accelerated can be calculated:

$$t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2 \cdot 0.008m}{65ms^{-2}}} = 0.0157s$$

The take-off velocity is thus;

$$v = a \cdot t = 65ms^{-2} \cdot 0.0157s = 1ms^{-1}$$

This, in turn, means that the jump height *h* is:

$$h = \frac{v^2}{2g} = 0.05m = 5cm$$

This result is not so unrealistic in comparison to the experimentally determined height of 3cm. If we take into consideration that part of the jumping energy flows off into the rotation, the value even seems to be quite plausible.

A similar somersaulting toy (a flipping mouse = "rat stuff", a word-play on "right stuff") was taken along into outer space as a mascot by American astronauts. There is a video showing some experiments with it in outer space [1]. The quality of the video pictures (NTSC system) is unfortunately deplorably bad.

The back somersault from a standing position belongs to the standard repertoire of good gymnasts. From *Fetz/Opavsky* [2], we have figure 4, which was drawn from a series of photos from a film camera. The upper row of numbers (50, 53, 54, and so on) indicates the number of the picture filmed. Since 64 pictures were filmed per second, the corresponding times can be calculated directly (lower row). For greater clarity, the pictures in the illustration were processed: in reality, the phases of flight would overlap. The body's center of gravity between picture 53 and picture 74 shifts only 40cm horizontally.

From [2], we can also gather that the body's center of gravity rises to *h* = 29cm during the phase of flight. If we assume the body's mass to be *m* = 75kg (unfortunately, the gymnast's mass is not explicitly specified in [2]. From the point of view of physics, sports literature is generally plagued by inadequacies.), the resulting potential energy is:

$$E_{pot} = m \cdot g \cdot h = 75kg \cdot 10ms^2 \cdot 0.29m = 225J$$

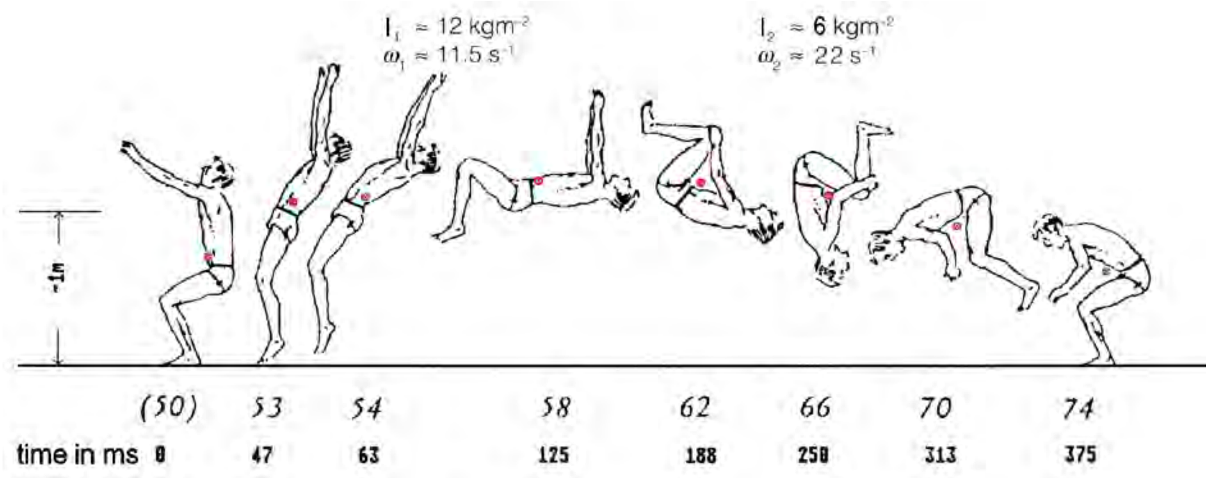

Fig. 4: Series of images of a back somersault from a standing position. The upper row of numbers indicates the number of the picture filmed, and the lower row shows the time in milliseconds (ms). The body's center of gravity is indicated by a red dot.

In order to calculate rotation energy, we need moments of inertia and angular velocity. Figure 4 shows the moments of inertia taken from *Hay* [3] for two body positions. The corresponding angular velocities for the phases of flight were calculated from relevant data in *Fetz/Opavsky* [2]. Approximate angular momentum can be confirmed by this, but in only rough approximation since some uncertainty accompanies estimations. For the rotation energy, for example, we get:

$$E_{rot} = \frac{I \cdot \omega^2}{2} = \frac{6kgm^2 \cdot 22^2 s^{-2}}{2} J = 1450J$$

The back somersault performed by a human can also be approached in a different way. The force with which a human pushes himself off is approximately *F = 1800N* [4]. It can also be estimated approximately that a human can lift a bit more that his own body weight (= 1000N) in addition to his own weight (750N). If this force had a direct effect on the center of gravity vertically above it, the result would be an acceleration of:

$$a = \frac{F}{m} - g = \frac{1800N}{75kg} - 10ms^{-2} = 14ms^{-2}$$

So, for the back somersault performed by a human, the main part of the energy is found in the rotation, whereas exactly the opposite is the case for the kangaroo. This is understandable since the potential energy is greater for the kangaroo because the jump height is relatively high in relation to the size of the animal's body. In addition, the kangaroo is built quite compactly, so that comparatively less energy is needed for the rotation.

This acceleration acts, for example, on a distance of *s = 30cm* (the body's center of gravity from the squatting to standing position). This results in an acceleration time *t* of:

$$t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2 \cdot 0.30m}{14ms^{-2}}} = 0.21s$$

This gives a take-off velocity *v* of:

$$v = a \cdot t = 14ms^{-2} \cdot 0.21s = 2.9ms^{-1}$$

and a jump height *h* of:

$$h = \frac{v^2}{2g} = 0.42m$$

This is more than for a back somersault experimentally since even more energy is put into the rotation during the somersault.

Suggestions for additional investigations that are not too detailed are, for example, the *torque moment, angular momentum, horizontal components in the somersault,* and the *parabolic flight trajectory of the center of gravity*. Perhaps you will even be interested in building your own somersaulting device.

The somersaulting toys with the wind-up motor are now (2013) available from http://zwindups.com

References:

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